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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Production leadtime (PLT) has experienced great fluctuations over the last few years. This can have a great effect upon the Inventory System of the Air Force. Presently the Air Force uses a forecasting model which assumes that PLT in the next time period is equal to PLT in the last time period. In order to improve upon this method, this study investigated both the use of ARMA and Smoothing Models for the prediction of PLT. The ARMA Model was eliminated as an effective prediction model for PLT. This was because the average number of data points

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was well below the minimum needed to make good forecasts using ARMA. An investigation of various smoothing models suggests that a significant improvement in forecasting can be achieved by using these models instead of the present Air Force Model.

AFOSR-TR- 80-1121

FINAL TECHNICAL REPORT

F49620-79-C-0170

THE STUDY OF PRODUCTION LEADTIME FORECASTING MODELS

PREPARED FOR:
THE AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
WASHINGTON, D.C.

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PURPOSE OF GRANT

During the mid-1970's, the United States' economy experienced a great increase in production leadtime. This had an adverse effect upon the inventory system of the Air Force in the form of frequent stock outs. The Air Force funded an ASEE Summer Faculty Research Project to investigate the area of production leadtime. The report for that project was submitted to both the Air Force Office of Scientific Research and XRS, of the Air Force Logistics Command, Wright Patterson AFB, Dayton, Ohio. This grant is an extension of that project, and it has two specific goals:

a. To develop specialized leadtime production models based on modifications of integrated autogressive moving average methods and exponentially weighted moving average methods.

b. To evaluate the models statistically to provide estimates of the prediction accuracy of current and proposed leadtime production methods.

BACKGROUND

Military Inventory Systems are extremely complex because of not only the number of items to be managed, but also the funding methods used to purchase these items. The Air Force has categorized its inventory into expendables (EOQ) and reparable items. The management of the entire inventory control system is the responsibility of the Air Force Logistics Command (AFLC), located at Wright Patterson Air Force Base in Dayton, Ohio.

The Logistics System, developdd by AFLC, has divided the United States into five Air Logistics Commands, centered in Ogden, Utah; Warner Robins, Georgia; Sacramento, California; Oklahoma City, Oklahoma; and San Antonio, Texas. These Air Logistics Commands act as wholesalers for both EOQ and reparable items. Each Air Logistics Command is responsible for a certain number of both EOQ and reparable items. Some ALCs specialize in certain types of items; for example, Warner Robins specializes in electronics.

At each ALC, item managers have the responsibility for from 100 to 1,500 items. Those item managers who have a small number of items typically handle expensive items or items that are subject to more frequent buys (known as high intensity items). For many years, the item manager kept track of items by hand, using rather simple inventory guidelines to keep proper stock levels. The Air Force then developed the D041, D062, J041, J014 requirements computation systems which provided two sources for the item manager. One system, the J041, provided history file of each EOQ item. From this system, an item manager could obtain various reports, such as last buy, costs, in-stock amounts, source, etc. The second system, D041, is a large EOQ computational system which determines the items that should be purchased and in what quantities. The input to that system is the J041. Within that model (D041), the assumption about both production leadtime (PLT) and administration leadtime (ALT) is that in the next period PLT and ALT will be equal to the prior period:

$$PLT_t = PLT_{t-1} \tag{1}$$

$$ALT_t = ALT_{t-1} \tag{2}$$

where:

PLT = Prediction of Production Leadtime

$PLT_{t=1}$ = Production Leadtime in Last Period

ALT = Prediction of Administration Leadtime

ALT_{t-1} = Administration Leadtime in Last Period

Thus both leadtimes are assumed to be known and constant. As one would expect, this assumption is often unrealistic, and therefore the effectiveness of the D041 is greatly limited.

Production and Procurement decided to attack this problem at two different levels: First, by developing a method which would update PLT on individual items; and second, by utilizing a model which could predict changes in PLT for industrial groupings. It was also hoped that at some time, the two models could be interfaced, possibly integrating them through the computer system D041 and D062, which initiates the buy orders. The first step taken by Procurement was the initiation of a PLT update system for all Air Logistics Centers (ALCs). It was hoped that the system would improve the present rule for updating PLT by using the actual PLT from the last buy (AFLC Regulation 57-6, 29 September 1977, ppg-3):

> Production Leadtime (PLT). Periods in time in whole months between date of contract or purchase order award and receipt of the first significant delivery quantity (under normal delivery conditions) based on the latest contract or purchase order or contract.

This information is manually inputed into the D042 (reparable items) and will soon be automatically inputed into the D062 system. Item managers can override this value if they have reason to believe that the PLT of the last buy is not a good indication of future PLT. The issue of using prior PLT as an estimate of future PLT is debatable and will be discussed later, but it seemed obvious to Procurement that the greatest errors in PLT would be on items that had not been purchased for a long period of time, since PLT can vary greatly, given various economic conditions.

Because of this, the first step taken was to develop a system to identify a list of items that would be in a buy position in the next year, and which also had not been purchased for the last six months. This list is automatically generated from the D041, D062, J041, and J013 system, and includes both reparable and EOQ items. The list is generated in March based on a 31 December computational process. A form letter is then sent out for each item to the last contractor, requesting an estimate of the present PLT for that item (see Appendix A). Contractors are under no obligation to respond to this letter, but the response has been about 70% for the last three years. These letters are then sent to item managers who are to change the PLT in the system (K/A file maintenance) if the PLT estimated by the contractor is substantially different from what is presently in the system.

The second step taken by Procurement in securing adequate estimates was the development of a predictive model for PLT which is independent of actual items purchased by the Air Force. The system started with the tracking average PLT as defined by Production Magazine for various industrial groups. This is updated on a monthly basis and is used to keep Procurement posted of any possible increases or decreases in PLT. In addition to the tracking of industrial groups, Procurement began the development of a regression model which used a series of independent variables, such as backlogs, new orders, etc., to predict PLT.

SURVEY OF LITERATURE

A great amount of research on leadtime has been undertaken both in and out of government. One typically defines leadtime by separating it into the production and administrative aspects of the purchasing process. Administrative leadtime is defined as the period of time from the existence of a demand until a purchase order is sent or a contract signed. From that time until the first significant delivery is received (at least 10%) is defined as production leadtime. Since the military has organizational control over administrative leadtime, more research has been done in this area than in the area of production leadtime. In order to gain an overall view of what has been done in the area of production leadtime, one can first look at some basic discriminating factors which have lead to various types of leadtime research. The bibliography lists a great many references to specific studies.

a. Base versus Air Logistics Command (ALC):

The most fundamental discriminating factor of PLT research is the level at which the procurement takes place. The Air Force orders items at two levels -- at the base and from Air Logistics Command. These two levels face entirely different problems. At the ALC, there are a number of item managers, each of whom has responsibility for a specific number of items.

At the base level, computation and implementation of PLT is entirely different. Each base takes the previous year's data for PLT on all items purchased (primarily locally purchased EOQ items) and uses this as the average PLT for all items. In addition, all items that had an actual PLT greater than 79 days are treated as outliers, and therefore not used in the computation of the average base PLT. This estimate of PLT is used for one year, at which time a new estimation is made based on the current year's data. The person administering the purchase is not an item manager, but a base supply manager. He has no way of overriding the calculated average PLT, even if his personal judgment indicates that the estimate is a poor one.

At most bases, an order is initiated automatically using some sort of classical EOQ base model, an input of which is the estimated PLT. This, of course, leads to extremely poor forecasting, since the items purchased have large deviations in production leadtime, yet the system assumes that each product has exactly the same leadtime. The above description of two operations should suggest the fact that the problems, and therefore research associated with the two levels, are extremely different. At the base level, the main concern is with identifying those items that traditionally have much longer PLTs than the base average for all items. Much of the research is focused on the shape of the distribution of PLT at the various bases. The second aspect of this research is identifying managerial methods of handling items that have traditionally had long PLTs. Current research in this area is pursuing a method which would have a series of PLTs that could be assigned to various item groups. It is believed that such a method would be more reflective of actual PLT than the present overall average method of prediction.

At the ALC level, one can assign unique PLTs for individual items. Therefore, the research has focused on methods of externally establishing the appropriate PLT and then assigning that value to individual items.

b. Economic Ordering Quantity versus Reparable Items:

Another difference in the procedure and research efforts is in the type of item purchased. The Air Force distinguishes between reparable items and expendables (known as EOQ). The item managers at the ALC facility are categorized in terms of their responsibility for either reparable or EOQ items. Further, AFLC has developed separate data systems, the D041 reparable and the D062 EOQ, for the two categories of items.

The importance of PLT in initiating buy orders seems to be greater for EOQ than for reparable items. This is due to the fact that the most sensitive parameters associated with buy orders for reparable items seem to be the average turnaround time on repairing items and the estimated condemnation rate.

By the same token, the reparable items typically have a much higher unit cost, and therefore, mistakes in estimated PLT can have significant effects upon the total buy dollars for AFLC. Another dimension of the dichotomy between these two groups is that the manager of reparable items typically has a better knowledge of each item, since he has fewer items to manage. Therefore, there is a greater opportunity to have some sort of computerized estimating procedure for the EOQ item manager than for the reparable item manager.

c. Tracking versus Predicting:

The next dichotomy in the type of research is the form of the model one would consider in estimating future PLT. One approach would be to examine models which simply smooth out variations in PLT history and, in turn, use this smoothed value as a predictor for future production leadtime. A second method would be to develop forecasts of future production leadtime based on a model which utilizes outside information such as economic data.

POSSIBLE SOLUTIONS AND METHODOLOGIES

There are a great number of approaches which can be used to predict production leadtime. Three methods were considered in this research:

a. The first involves utilizing a model which predicts PLT based on external factors, such as the economy. This was attempted by Westinghouse, who had used a basic regression model to predict the PLT of hot and cold rolled steel. This model met with mixed results. A similar, but more extensive, attempt at predicting PLT using external data was made in this research grant during the Air Force Summer Faculty Program at AFLC, Wright Patterson. These models are useful at a managerial level where overall categories of products can be observed for changes in PLT. On the other hand, disagregation of the industrial groups into specific items in the D041 is impossible.

b. A second approach is to view PLT as a random variable. With such an assumption, one would first try to estimate the form of the distribution of PLT. Once the distribution was established, there would be two choices. The first would be to utilize some form of a traditional safety stock leadtime model. This method will typically lead to conservative estimates for PLT. The second choice would be to incorporate production leadtime explicitly into a compound distribution of leadtime demand. This approach has been suggested by Dr. Jack Hayya of the Pennsylvania State University. Appendix B is a paper of this approach which will be presented at the Western AIDS Meeting this March. It is based on the leadtime data base used in this study.

c. The final method is to treat production leadtime as a time series to be predicted. This method is the one that this grant was to investigate. Possible models that could be used would be either auto-regressive or smoothing, or some combination of the two.

RESULTS

As indicated above, this grant was projected to consider both ARMA and smoothing techniques to predict production leadtime. Our assumption was that we would have a reasonable number of observations of production leadtime for the items we investigated. As will be discussed in detail in the "Data Section" below, the data which was secured averaged about 20 observations per item. In addition, these items were not typical of the overall inventory because they are all high intensity items. Although no formal statistical estimations were made, it is safe to say that the typical number of PLT observations for EOQ items is less than 10. This finding was of course a great disappointment since ARMA models are not highly useful with so few observations. In particular, Box-Jenkins methodology requires at least 30 observations to be effective (Box, Jenkins). A second negative factor was the fact that ARMA models demand some level of interaction on the part of the user. Since the number of EOQ items that the Air Force buys is in the hundreds of thousands, we decided that the use of an ARMA model would not be practical for the predicting production leadtime of individual EOQ items.

Therefore, it was decided that the most practical solution would be to fully investigate all possible smoothing methods. Such methods are excellent for problems where there are a large number of items, and there are no minimum number of observations needed to utilize these techniques.

MODELS

Five models were tested to predict production leadtime. A description of each of these models and its nature follows:

Air Force Model

As noted above, the Air Force uses the latest PLT as a prediction of future PLT.

$$F_{t+1} = X_t$$

where:

F_{t+1} = Estimate of PLT in Next Period

X_t = Actual PLT in Current Period

Such a model is known as a Naive Forecasting 1 Model by Makridakis and Wheelwright. They suggest its use as a base for evaluating alternative forecasting methods.

Single Exponential Smoothing

Single exponential smoothing is the most basic exponential smoothing technique. It allows the user to make a choice between the importance of current versus past data. This is accomplished by the selection of the alpha weight. The closer alpha is to one, the more weight will be given to current data. The model is shown below:

$$F_{t+1} = \alpha X_t + (1-\alpha)F_t$$

where:

F_{t+1} = Estimate of PLT in Next Period

X_t = Actual PLT in Current Period

F_t = Forecast Made for Current Period

α = Alpha

If the above model is expanded, then it becomes:

$$F_{t+1} = X_t + \alpha(1-\alpha)X_{t-1} + \alpha(1-\alpha)^2X_{t-2} + \alpha(1-\alpha)^3X_{t-3}$$

$$+ (1-\alpha)^4X_{t-r} + \alpha(1-\alpha)^5X_{t-5} + \ldots\ldots + \alpha(1-\alpha)^{N-1}X_{t-(N-1)}$$

It has been found that an alpha of .10 to .20 is best for most inventory applications. The closer alpha is to zero, the more smoothing will take place and vice versa.

Linear Exponential Smoothing

Brown's Linear Exponential Smoothing allows for a trend to be included in the forecast. Makridakis and Wheelwright point out that "the underlying rationale of Brown's Linear Exponential Smoothing is similar to that of linear moving averages: since both the single and double smoothed values lag the actual data when a trend exists, the difference between the single and double smoothed values can be added to the single smoothed value and adjusted for trend." Brown's model appears below:

$$F_{t+m} = \alpha_t + b_t m$$

where:

$$\alpha_t = S'_t + (S' - S''_t) = 2S'_t - S''_t$$

$$b_t = \frac{\alpha}{1-\alpha} (S'_t - S''_t)$$

$$S'_t = \alpha X_t + (1-\alpha) S'_{t-1} \qquad \text{[Single Smoothed]}$$

$$S''_t = S'_t + (1-\alpha) S''_{t-1} \qquad \text{[Double Smoothed]}$$

Quadratic Exponential Smoothing

This model is similar to the linear model except that the trend is quadratic. The formulas are more complex, and the results will be erratic if the data does not follow a quadratic shape. The formulas for Brown's Quadratic Exponential Smoothing are shown below:

$$F_{t+m} = \alpha_t + b_t m + \tfrac{1}{2} C_t m^2$$

where:

$$\alpha_t = \alpha S''_t + (1-\alpha)\ S'''_{t-1}$$

$$b_t = \frac{\alpha}{2(1-\alpha)^2}\ [(6-5\alpha)S'_t - (10-8\alpha)S''_t + (4-3\alpha)S'''_t\ ]$$

$$C_t = \frac{2}{(1-\alpha)^2}\ (S'_t - 2S''_t + S'''_t)$$

$$S'_t = \alpha X_t + (1-\alpha)\ S'_{t-1}$$

$$S''_t = \alpha S'_t + (1-\alpha)\ S''_{t-1}$$

$$S'''_t = \alpha S''_t + (1-\alpha)\ S'''_{5-1}$$

<u>Adoptive-Response-Rate Single Exponential Smoothing</u>

The final model is an extension of the single exponential smoothing model. This model does not require the choice of an alpha value, but rather allows alpha to change on an ongoing basis. In essence, the data itself will automatically generate the appropriate alpha value. The formulas for this model are shown below:

$$F_{t+1} = \alpha_t X_t + (1-\alpha_t) F_t$$

where:

$$\alpha_{t+1} = \left| \frac{E_t}{M_t} \right|$$

$$E_t = \beta e_t + (1-\beta) E_{t-1}$$

$$m_t = \beta |e_t| + (1-\beta) M_{t-1}$$

$$e_t = X_t - F_t$$

$$\beta > 0 \text{ and } < 1$$

The beta value is a measure of the adjustment to be done on alpha. In the past beta values between .10 and .20 have proven most successful.

DATA SOURCES

One of the most difficult aspects of this project was the securing of the data set. The project began at Wright Patterson by requesting production leadtime data from a variety of sources. None of those requested could secure such data. It therefore became evident that good data could only be secured at the ALC level. A second factor that became evident was that many of the items that the Air Force procured involved only a few buys. Thus, it would be difficult to secure items which would have enough history on production leadtime to generate accurate data analysis. To solve this problem, the research examined only high intensity items. These are items that are of particular importance to the Air Force and are typically purchased on a relatively frequent basis. Each ALC has approximately 200 high intensity items.

It was decided to use the high intensity items from Warner Robins. XRS made arrangements for a visit to that ALC, and after a great effort, data was secured for about 50 items. In addition to PLT, the data has some additional characteristics, such as cost, quantity, etc. These were recorded in the hopes that others might investigate some possible relationships between PLT and some of these characteristics. The author of this report expects to do some independent analysis of these characteristics, and would be happy to make the data set available to AFSOR if they feel that others might have use for it.

DATA RESULTS

The data was tested using the five models described above. The Air Force model (A/K/A Naive Model 1) was used as a base to evaluate the four smoothing models tested. In order to make a judgment about the best model, the two most common measures of forecasting accuracy were used. These methods are shown below:

Mean Squared Error (MSE)

$$MSE = \frac{\sum_{i=1}^{n} (X_i - F_i)^2}{n}$$

where:

X_i = Actual Value

F_i = Forecast Value

n = Number of Data Values

Although MSE is a common method of evaluating forecasting methods, it has a number of drawbacks. The first is that one can always get a low MSE by utilizing a higher order forecasting method which gets a good fit on old data, but which might not be appropriate for future forecasts. A second problem is that any specific MSE value has little meaning in terms of describing the level of accuracy of the forecasting errors.

Because of these deficiencies, some have turned to the Mean Absolute Deviation Error (MADE) as a way to make judgments about the accuracy of various forecasting methods. The formula for this method is shown below:

$$MADE = \frac{\sum_{i=1}^{n} \left| X_i - F_i \right|}{n}$$

where:

X_i = Actual Value

F_i = Forecast Value

n = Number of Data Values

The data set described above was imputed into a program which utilized all five forecasting models described above. The program used was provided by Dr. Jack Hayya of the Pennsylvania State University. In that PLT data has a high level of variation, the MADE was chosen as a criteria for forecasting accuracy. Table 1 shows that the average MADE for 21 items was 71.16 using the Air Force Model. For each of the other four models, three MADE values are shown. The forecasting program used tests for the best MADE by ranging with an alpha from .10 to 1.00 by units of .09. Table 1 shows the lowest MADE value associated with the various alpha values. In addition it shows the MADE values associated with .10 and .19 alpha. The final column of Table 1 shows how often any model provided the best prediction accuracy, based on the lowest MADE.

TABLE 1
COMPARISON OF FORECASTING BY AVERAGE MADE
(21 Items)

	α	MADE	Best Model
Air Force		71.16	
Single	Best	54.09	9
	.10	59.47	0
	.19	57.95	2
Linear	Best	54.80	2
	.10	58.28	1
	.19	60.28	0
Quadratic	Best	55.61	1
	.10	59.66	1
	.19	65.28	0
Aress	Best	59.52	5
	.10	64.76	0
	.19	64.66	0
			21

Viewing Table 1 we can see that the Single Smoothing Model using the best alpha has the lowest average MADE (54.09) and was the best predictor. The Linear Model using the best predictor was not far behind with an average MADE of 54.80. In viewing the models that the user picks a specific alpha (beta for ARRESS) the best model appears to be Single, using .19 as the alpha value.

These results, along with some additional analysis of the other generated data are now being formulated into an article "The Study of Production Leadtime Forecasting Models." This will be submitted to a journal such as <u>The Journal of American Production and Inventory Control Society</u>.

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List of Publications

1. Cleary, Michael J. "Forecasting Production Leadtime," Proceedings, Tenth Annual Meeting on Simulation Modeling, pgs 517-521, Pittsburgh, Penn., April 1979.

2. Cleary, Michael J.; Hayya, Jack; Gross, Paul W. "Setting Reorder Levels With Stochastic Leadtime and Demand," Proceedings, Midwest AIDS, April 1980, Dayton, Ohio.

3. Cleary, Michael J.; Hayya, Jack C.; Schaul, Ronny A. "Approaches to Treating Leadtimetime in Establishing Inventory Service Levels," Proceedings, Western AIDS, March 1981, Hilo, Hawaii.

4. "The Study of Production Leadtime Forecasting Models," this article will report on the final results of the comparison of the five forecasting methods. I will submit it to an application oriented journal, such as The Journal of American Production and Inventory Control Society.

Interactions (Coupling Activities)

1. During the period of this grant, I have been in contact with AFLC/LORRA at Wright Patterson AFB. I have kept them appraised of my progress on the leadtime study. Mrs. Carol Hawks has been my main contact person. In addition, I have worked closely with Dr. Jack Hayya of the Pennsylvania State University who had a contract to investigate leadtime demand for Air Force Business Research Management Center.

2. Cleary, Michael J. and Hayya, Jack C. "Leadtime--Its Effect Upon Military Inventory Control," paper presented to the TIMS/ORSA Joint National Meeting, Washington, D.C., May 1980.

APPENDIX A

DEPARTMENT OF THE AIR FORCE
Headquarters, Air Force Logistics Command
Wright-Patterson Air Force Base, Ohio 45433

AFLC REGULATION 84-4

28 July 1976

Production

PRODUCTION LEADTIME ACQUISITION

This regulation sets forth responsibilities and provides guidelines to the Air Logistics Centers (ALCs) for identifying current production leadtimes for use in requirements computations.

1. General. The Air Force Logistics Command worldwide mission requires that aerospace forces be provided logistics support, materiel, and services. Achievement of the mission requires careful planning, scheduling, and management in the development of materiel requirements. Such requirements are determined by the materiel management function. Procurement and production provides the necessary procurement support to plan for and obtain items required for mission programs. To assure effective and timely determination of requirements, materiel management requires accurate estimates of production leadtimes (PLT). PLT is continually subject to change, depending on numerous factors. Requirements computation methods generally rely on historic PLT; for example, leadtime experienced on the latest buy. On occasion, recent contractual quotes, PLT for similar items, and current contractor estimates are used. The accuracy of PLT maintained affects the determination of realistic delivery schedules and ultimately the ability of contractors to meet those schedules and fill command needs.

2. Responsibility. The Directorate of Procurement and Production (D/PP), Procurement Planning and Technical Support Branch (PPDM) is the office of primary responsibility (OPR) within each ALC for the acquisition of current PLT (except at Ogden ALC where the D/PP will designate the OPR). Upon request, PPDM will obtain current PLT for specific National Stock Numbered items identified by the Directorate of Materiel Management (D/MM), Requirements Branch (MMMR).

3. Program Operation:

a. The primary means of determining current PLT estimates is by direct written or verbal contact with the sole source or most recent contractor. Contractor response to such a request is to be voluntary and at no cost to the Government. Written requests are the preferred means of contact as they can better identify to the contractor the need for and mutual benefits to be derived from current PLT. All written requests will be made, using a letter similar in content to that of the sample letter included as an attachment hereto.

b. This regulation will be implemented through the use of the semi-automated Production Leadtime Survey. This survey utilizes an annual product generated from data in the D041, D062, J041 and J014 systems. It identifies recoverable and economic order quantity items projected to be in a buy position during the budget year that do not have a procurement action in process or completed in the last six months, or are otherwise screened by programmed edit routines. The product will normally be run in the month of March based upon the 31 December computation cycle. MMMR will forward the survey printouts, grouped in contractor sequence to PPDM for processing the survey to contractors. Computer generated contractor address labels will be forwarded directly to PPDM by AC.

c. PPDM will attach a form letter to printouts applicable to each contractor. The computer printed address labels will be used to the greatest extent possible on the form letters and envelopes for mailing the survey to contractors. In those instances where an address label is not available for a contractor, PPDM shall obtain the address. Survey printouts will be forwarded to contractors within 10 workdays after receipt from MMMR.

d. Success in accumulating realistic PLT is dependent on close cooperation with the contractor. Contacts with contractors shall emphasize that response is voluntary. Any PLT received should be in the form most convenient to the contractor. However, the survey printout is intended for use by contractors as a turnaround document for their convenience. A return envelope addressed to PPDM will be enclosed in the package sent to the contractor. Replies received by PPDM will be forwarded to MMMR within 5 workdays. If PLT for any item appears unreasonable, the IM may identify that item to PPDM for further investigation and research.

e. For those items that the IM determines the contractor quoted PLT should be entered into the requirements computation, the following time standards for file maintenance action apply: EOQ items—two weeks; recoverable items—during next quarterly computation cycle.

f. Occasionally the IM may decide that an item not included in the survey needs PLT update. In this case he should forward his request through channels to PPDM. The request should contain information substantially the same as in the survey printout (for example, NSN, supplier or MFR code, part number, item name, PLT, IM code, etc). PPDM will forward this information to the contractor as

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Supersedes AFLCR 84-4, 14 Mar 74. (For summary of revised, deleted, or added material, see signature page).
OPR: PPMP
DISTRIBUTION: X

AFLC-WPAFB-AUG 76 150

an attachment to the form letter described in paragraph 2a. The contractor reply should be forwarded to MMMR in accordance with paragraph 3d—time standards set forth in paragraphs 3c, d and e are applicable to these items.

g. Should a contractor be unable to provide current PLT, or direct contact with a contractor is determined to be impracticable, a best estimate should be provided, upon IM request, based on item analysis and comparisons with leadtime for similar items, trend information available in industry trade publications, or information from other Government activities. When such estimates are given, MMMR shall be advised of the basis for same.

h. The Contract Administration and Operations Branch (PPDO or at Ogden ALC, the Contract Management Branches, PPZC and PPSC) may occasionally discover PLT changes through its production surveillance function. Significant changes should be referred to PPDM for research and forwarding of current PLT to MMMR. Updating such information may help to avoid future contract production problems by assuring more realistic schedules. Referral of such changes should, however, be restricted to unusual cases; for example, where it is known that historic PLT will not be updated for several months.

4. Program Control. PPDM will use a carbon copy of the survey printout as a control register to annotate progress and final disposition of each individual or collective (in the case of a list of items) request resulting from the semi-automated PLT survey. Items manually identified by IMs for update action will be recorded and controlled by a separate log.

5. Reporting. The Production Leadtime Survey Report has been created to measure the fluctuation of production leadtimes and to assess the value of the production leadtimes acquisition program. The report has been assigned Report Control Symbol RCS: LOG-PP(A)-7601. The report will be forwarded by mail to AFLC/PPMP with a copy to MMRRS by 30 June, with data as of 15 June. The following information will be contained in the report:

1. Number of Items Identified for the Survey
2. Number of Items Sent to Contractors
3. Number of Contractors Queried
4. Number of Items Returned by Contractors
   - A. Number with Increased PLT of:
     - (1) One Month
     - (2) Two Months
     - (3) Three Months
     - (4) Four or More Months
   - B. Number with Decreased PLT of:
     - (1) One Month
     - (2) Two Months
     - (3) Three Months
     - (4) Four or More Months
   - C. Number Unchanged
5. Number of Contractors Responding
6. Number of Items Receiving File Maintenance Actions.
   - A. Number with Increased PLT
   - B. Number with Decreased PLT

F. M. ROGERS, *General, USAF*
Commander

C. W. MORIN, *Colonel, USAF*
Director of Administration

1 Attachment
Sample Letter

Summary of Revised, Deleted, or Added Material

The PLT Acquisition program is revised to specify the Semi-Automated PLT Survey as the nucleus of the program in lieu of manual techniques. Also, Time Standards for processing the PLT survey and an RCS rep. ˢing requirement are added.

DISTRIBUTION: X

HQ USAF/LGPMA	2
AFISC/DPAL and AUL/[illegible]	2 ea
HQ AFLC	4
(PPMP.....1; P[illegible]-3A.....1; IG.....1; MMRRS.....1)	
ALCs (PPDM.....5; MMMR.....5; DA.....2)	16 ea
2750/DAPL	3

SAMPLE LETTER

From: ...... ALC-PPD

Subject: Request for Production Planning Information

To: (Contractor)

1. Air Force Logistics Command equipment and spare part buy determinations are based primarily on procurement experience with indivdual items. To assure effective procurement planning, it is essential to have accurate estimates of production leadtimes. Accurate estimates also benefit the supplier of items by assuring realistic delivery schedules. Inventory managers, therefore, periodically select items which require production leadtime update.

2. Our records indicate that you have previously furnished (National Stock Number, Nomenclature) to the Air Force. (Note: for a number of items, substitute: Attached is a list of items, by Federal Stock Number and Nomenclature, which our records indicate you have previously furnished the Air Force.) To assist in our planning, we would appreciate your providing current estimates of production leadtimes; for example, that time from receipt of an order to shipment of first production units, assuming an economic production run. The estimates should assume either a follow-on order, where your experience indicates periodic Government orders and continued production, or an initial order, where an item has been infrequently or irregularly purchased.

3. Current production leadtimes may be provided in whatever form you desire; however, the attached list was designed for your convenience to be annotated and returned in the envelope provided.

4. This request is strictly for planning purposes and response is entirely voluntary. It should not be considered as an indication that a procurement of the identified items is forthcoming or contemplated, or that the Government intends to pay for the information. Sole purpose for the request is to obtain accurate leadtimes which can serve our mutual interests.

5. Should you desire additional information regarding this request, please contact ....

APPENDIX B

APPROACHES TO TREATING LEADTIME IN ESTABLISHING INVENTORY SERVICE LEVELS

Jack C. Hayya, The Pennsylvania State University
Michael J. Cleary, Wright State University
Ronny A. Schaul, Bureau of Labor Statistics

Paper submitted for presentation in the Area of Operations Analysis (Inventory Models and Systems) at the Western AIDS Conference, Hilo, Hawaii, March 18-21 and 23-24, 1981.

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APPROACHES TO TREATING LEADTIME IN ESTABLISHING INVENTORY SERVICE LEVELS

ABSTRACT

More and more, industrial firms and governmental procurement agencies are encountering shortages in stocks precisely because their inventory systems are not geared to handling the variability of leadtime. This paper surveys the current approaches, using examples from a recent study at the US Air Force Logistics Command (AFLC). Coming to grips with leadtime variability implies, in general, larger order quantities, more investment in buffer stocks, and higher annual operating costs.

I. INTRODUCTION

There are four approaches to treating leadtime: 1) as a constant; 2) as perfectly autoregressive, where the next observation of leadtime is forecast to be equal to the previous one; 3) as a safety leadtime, where some upper percentage point of the distribution of leadtime is used; and 4) as explicitly incorporated in the compound distribution of leadtime demand. The first two of these approaches are quite unsatisfactory; the third may be quite conservative; and the fourth is theoretically the proper approach.

Leadtime Constant

Consider the following actual time series of the procurement leadtime in days for an electron tube:

$$L = \{147, 42, 581, 249, 480, 695, 58, 71, 104\}. \quad (1)$$

For such a small sample, the exponential, lognormal, Weibull, gamma, and normal distributions produce good fits, in the order given. Suppose we wished to use the mean or the median as a point estimate for leadtime. The mean $\bar{L}$ = 270 days and the median m_L = 147 days. For the item in question, daily demand is estimated at E(D) = 3.4 units, and using the mean or the median for leadtime would not affect buffer stock if the organization adheres, as many do, to a policy of a two or a three-months supply. The choice of an estimate for leadtime would, however, affect the reorder point, the number of back-orders and the holding costs.

Perfect Autoregression

In this approach we assume that the next leadtime is equal to the previous one, that is,

$$\hat{L}_t = \hat{L}_{t-1}. \tag{2}$$

In 1965, Fama [6] postulated (2) as an excellent model for predicting the behavior of stock prices, but what the model (2) requires is a high positive correlation between adjacent observations. We have not found this to be true for any item in a sample of size n = 61 taken at AFLC; and in applying the runs test (Siegel [11, pp. 53-56 and pp. 252-253]) for the items, we could accept the hypothesis of randomness by a very comfortable margin.

Practitioners are belatedly coming to the realization that this approach is quite unsatisfactory for the prediction of leadtime. In a study for the U. S. Army, for example, Cohen [3] points out that the mean dominates the latest value as an estimate for leadtime, the mean being unbiased, minimum variance, asymptotically normal, etc. There are, of course, some sophisticated techniques of robust estimation for location, but a discussion of these is beyond the scope of this paper.

Safety Leadtimes

Hadley and Whitin [7] recommend the use of maximum leadtime, max(L), or $\bar{L} + \sigma(L)$. From (1), max(L) = 695 days, whereas $\bar{L} + \sigma(L) = 520$, because $\sigma(L) = 250$. In an MRP context, Whybark and Williams [13] recommend $\bar{L} + k\sigma(L)$, $k > 1$. If we supposed that leadtimes are normally distributed, then a ninety-five percent safety leadtime would be 681 days.

Compound Distributions

The appropriate approach is to think of leadtime demand as a random sum of random demands that are independently and identically distributed. Leadtime demand may be written as

$$X = D_1 + D_2 + \ldots + D_i + \ldots + D_L, \quad i = 1,2,\ldots,L, \tag{3}$$

where D and L are random variables denoting demand and leadtime. Thus leadtime demand, X, may be thought of as a mixture, while leadtime, L, is the mixing distribution. More specifically, f(X) may be said to be a compound distribution with G(L) being the compounding distribution (Ord [9, pp. 64-66]).

It can be shown (Drake [5, pp. 109-112]) that for the structure (3),

$$E(X) = E(L)\cdot E(D). \tag{4}$$

and

$$V^*(X) = E(L)\cdot V(D) + [E(D)]^2\cdot V(L), \tag{5}$$

the star denoting the variance of leadtime demand with variable leadtime.

If, on the other hand, leadtime is constant at L, then E(X) would be as in (4) but

$$V(X) = E(L)\cdot V(D). \tag{6}$$

We can immediately see the influence on safety stocks if we begin to consider the variability of leadtime. For the same safety factor, k, this increase would be in the ratio

$$\frac{\sigma^*(X)}{\sigma(X)} = \left(\frac{E(L)V(D) + [E(D)]^2 V(L)}{E(L)V(D)}\right)^{1/2}$$

$$= \left(1 + \frac{VMR(L)}{VMR(D)} \cdot E(D)\right)^{1/2} \text{, where VMR denotes the variance to mean ratio} \qquad (7)$$

$$\approx \sigma(L)\left(\frac{E(D)}{E(L)}\right)^{1/2} \text{, for } VMR(D) = 1 \text{ and large } VMR(L).$$

Theory of Compound Distributions

One of the best surveys on the role of compound distributions in inventory theory will be found in McFadden [7], and it would be relatively easy if compound distributions were theoretically tractable; sometimes they are, sometimes they are not. Hadley and Whitin [7, p. 117] have shown that where the procurement leadtime is gamma-distributed with parameters α, β, and if a Poisson process with mean λt generates demands with units being demanded one at a time, the distribution of leadtime demand is a negative binomial with parameters $\alpha + 1$, $\beta/(\beta + \lambda)$. Burgin [3] has treated the case with demand normal and leadtime gamma. There is other work (for example: Sherbrooke [10]; Ord [9]; Bott [1]), but the leadtime distributions are too complex for the ordinary practitioner.

II. A COMPARISON OF THE FOUR APPROACHES

Using the data in (1), we give in Table 1 a comparison of these approaches of using leadtime. We assume, as frequently done in practice, a safety stock equivalent to a two-month supply, and we calculate the reorder point and the holding cost applied to the inventory position. The reorder point is simply

TABLE 1

EFFECT OF APPROACH ON REORDER POINTS AND HOLDING COSTS (ELECTRON TUBE)

Approach	Mean Leadtime Demand $\mu = E(D) \cdot L$	Buffer Stock, B	Reorder Point $P = \mu + B$	Holding Costs[1] $HC = ac(\mu + B)$
Constant Leadtime: $\bar{L} = 270$	918	216	1134	$23,077
$m_L = 147$	500	216	716	14,571
Latest Value: $L = 104$	354	216	570	11,600
Maximum Value: $L = 695$	2363	216	2579	52,482
$\bar{L} + \sigma(L)$: $L = 520$	1768		1984	40,375
$\bar{L} + 1.64\,\sigma(L)$: $L = 680$	2312	216	2528	51,445
Compound Distr., Normal Approximation, 95% Level	2306	216	2522	51,323

LEGEND:

$E(D) = 3.4$ units per day
$\sigma(L) = 250$ days
$B = (3.4)60 = 216$ units
$a = 0.25$
$c = \$81.40$

[1]The calculation ignores the quantity Q/2.

$$
\begin{aligned}
P &= \mu + B \\
&= E(D)\cdot L + B,
\end{aligned}
\tag{8}
$$

where μ is mean leadtime demand and B represents buffer stock. The holding cost applied to the inventory position is

$$HC = ac(\mu + B + Q/2), \tag{9}$$

where a is the holding cost factor, c is the unit cost, Q is the order quantity, and μ and B as before. For simplicity we shall ignore Q/2 in the present discussion, that is, we shall use

$$HC = ac(\mu + B). \tag{10}$$

Inspecting Table 1 we see that depending on the approach, the reorder point fluctuates dramatically. This has implications in terms of annual holding cost as can be seen. Furthermore, the impact in terms of shortages will, of course, be more severe the lower the reorder point.

A SIMPLE MATHEMATICAL FORMULATION

The following formulation is similar to Bramson's [2], except that we have carried it out farther and assumed Poisson daily demand rates and a normal approximation for leadtime demand in the right tail. This latter assumption is recommended by Wagner [12], and we have actually verified it by simulating Poisson daily demands with normal, lognormal, and gamma leadtimes.

In the discussion below, we may use $\overline{L}$, $\overline{D}$ interchangeably with E(L), E(D). Suppose that leadtime is constant at $\overline{L}$. Suppose that demand is stochastic with mean E(D) and variance V(D). Let demand during leadtime be X. Then

$$E(X) = \bar{L}\, E(D), \tag{11}$$

$$V(X) = \bar{L}\, V(D). \tag{12}$$

This is because of the structure (3). But with L variable, E(X) will remain as in (11), but V(X) will be as given in (5).

In general, the multiplier effected in buffer stocks by considering variable leadtime is

$$\begin{aligned} \eta &= \left\{1 + \frac{[E(D)]^2 V(L)}{\bar{L}\, V(D)}\right\}^{1/2} \\ &= \left\{1 + \frac{V(L)}{\bar{L}} \frac{1}{[\text{v.c.}(D)]^2}\right\}^{1/2}, \end{aligned} \tag{13}$$

where c.v. stands for the coefficient of variation. If the demand is Poisson-distributed, this can be simplified to

$$\eta = \left\{1 + \frac{\bar{D}\cdot V(L)}{\bar{L}}\right\}^{1/2}. \tag{14}$$

As for the increase in the number of units required to maintain a given service level, this will be according to the ratio

$$\begin{aligned} \psi &= \frac{\bar{L}\cdot E(D) + k\{\bar{L}\cdot V(D) + [(E(D)]^2\cdot V(L)\}^{1/2}}{\bar{L}\cdot E(D) + k\{\bar{L}\cdot V(D)\}^{1/2}} . \\ &= \frac{1 + k\left\{\dfrac{\bar{L}.V(D) + [E(D)]^2 V(L)}{\bar{L}^2[E(D)]^2}\right\}^{1/2}}{1 + k\left\{\dfrac{\bar{L}.V(D)}{\bar{L}^2[E(D)]^2}\right\}^{1/2}} \\ &= \left\{1 + k\sqrt{\frac{1}{\bar{L}}[\text{c.v.}(D)]^2 + [\text{c.v.}(L)]^2}\right\} \Big/ \left\{1 + k\frac{\text{c.v.}(D)}{\sqrt{\bar{L}}}\right\} \end{aligned}$$

$$= \left\{1 + k\sqrt{\frac{1}{\overline{DL}} + [c.v.(L)]^2}\right\} / \left\{1 + k/\sqrt{\overline{DL}}\right\}, \text{ for Poisson demands}$$

$$\approx \frac{1 + k\ c.v.(L)}{1 + k/\sqrt{\overline{DL}}}, \text{ for moderate } \overline{DL}, \qquad (15)$$

$$\approx 1 + k\ c.v.(L), \text{ for very large } \overline{DL}, \qquad (16)$$

where k is the safety factor. The amount k c.v(L) represents the upper limit of the percentage increase in stock levels or investment.

Example. For electron tube, the estimated daily demand rate is $\overline{D} = 3.39$. Leadtime has mean $\overline{L} = 270.10$ and standard deviation $\sigma(L) = 249.98$. The mean of leadtime demand would be

$$\begin{aligned} E(P) &= \overline{L}\ E(D) \\ &= 270.10\ (3.39) \\ &= 915.6 \text{ units.} \end{aligned}$$

With leadtime assumed constant,

$$\begin{aligned} V(X) &= \overline{L}\ V(D) \\ &= 270.10\ (3.39) \\ &= 915.6, \end{aligned}$$

and

$$\sigma(X) = 30.26.$$

But with leadtime variable,

$$\begin{aligned} V(X) &= \overline{L}\ V(D) + [E(D)]^2 V(L) \\ &= 915.6 + (3.39)^2 (249.98)^2 \\ &= 915.6 + 718141.3 \\ &= 719056.9, \end{aligned}$$

and

$$\sigma^*(X) = 849.97.$$

The increase in buffer stock is by a factor of

$$\eta = \frac{\sigma^*(X)}{\sigma(X)} = 28,$$

and, hence, the percentage increase is 2700 percent. The 97.5 percent service level with leadtime constant is

$$\begin{aligned} X_{0.95} &= E(X) + 2\sigma(X) \\ &= 915.6 + 2(30.26) \\ &= 976 \text{ units.} \end{aligned}$$

But with leadtime variable, it is

$$\begin{aligned} X^*_{0.95} &= E(X) + 2\sigma^*(X) \\ &= 915.6 + 2(849.97) \\ &= 2616 \text{ units.} \end{aligned}$$

This increases the 97.5 service level by a factor of 2.68, or by 168 percent. Note that c.v.(L) = 0.9255 and that the above result could be directly obtained from equation (15).

Examples of the percentage increase in buffer stocks and in 95 percent service level stocks are tabulated in Table 2. It may be instructive to calculate the increase in inventory investment that may be required in taking account of variability in leadtime. For the electron tube, for example, raising the service level from 976 to 2616 requires an additional investment of $81.40 (2616 - 976) = $133,415. The increase in investment for that part would be about 168 percent.

TABLE 2

PERCENTAGE INCREASE IN BUFFER AND SERVICE LEVEL STOCK
WITH VARIABLE LEADTIME

FSN: Noun	1978 Unit Price	1978 Daily Demand, $\overline{D}$	Leadtime t	$\sigma(L)$	c.v.(L)	Buffer Stock	97.5% Service Level: Increase in Stock
58.....4611: Chopper File	$57.61	1.60	379	63.6	0.17	320%	24%
589....4403: Control	$239.00	3.80	284	108.3	0.38	1155%	76%
589....508: Amplifier	$250.00	2.49	262	96.1	0.37	842%	61%
596....106: Electron Tube	$81.40	3.39	270	250.0	0.93	2700%	168%

Implications. Example: Electron Tube

An additional investment of $134,000.

SUMMARY AND CONCLUSIONS

This paper reports on a study of the effect of leadtime variability on safety stocks and service levels. The study uses data from the US Air Force Logistics Command. A simple mathematical formulation, using Poisson daily demands and a normal approximation for the right tail of the distribution of leadtime demand shows that the increase in stock levels required with leadtime variability explicitly incorporated is bounded by an upper limit kc.v.(L), where k is the safety factor and c.v.(L) is the coefficient of variation of leadtime.

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